

## Exercise 77

For what values of  $a$  and  $b$  is the line  $2x + y = b$  tangent to the parabola  $y = ax^2$  when  $x = 2$ ?

### Solution

Rewrite the equation for the tangent line.

$$y = -2x + b$$

We see that the slope is  $-2$ . Take the derivative of the equation for the parabola.

$$y' = \frac{d}{dx}(ax^2) = a \frac{d}{dx}(x^2) = a(2x) = 2ax$$

When  $x = 2$ , the derivative must be equal to  $-2$ .

$$y'(2) = 2a(2) = -2 \tag{1}$$

The tangent line intersects the parabola at  $x = 2$ , so the equations that represent them are equal when  $x = 2$ .

$$\text{At } x = 2: \quad ax^2 = -2x + b \quad \rightarrow \quad a(2)^2 = -2(2) + b \tag{2}$$

Solve equations (1) and (2) for  $a$  and  $b$ .

$$a = -\frac{1}{2} \quad b = 2$$

Therefore,  $2x + y = 2$  is tangent to the parabola  $y = (-1/2)x^2$  when  $x = 2$  as the following figure illustrates.

